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Resistant performance of perforation in protective structures using a semi-empirical method with marine applications

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Abstract

To predict the performance of protective structures, in a preliminary design for weapons effects analysis processes considering the ship vulnerability evaluation, it is important to choose a method which is simple, quick and feasible. This paper presents a semi-empirical method to study the resistant performance of perforation in protective structures for fragment effects, such as residual velocity after penetration of structures between source of explosion and the target. In the calculation procedure, the mass distribution of fragments is obtained by expressing Mott's equation, the initial and striking velocities are calculated using Gurney's formulas and the residual velocity is estimated using Baker et al.'s equation. Using experimental results from the literature, Woodward's test data (1978) and Edwards and Mathewson's experimental data (1997) are employed to verify the ballistic limit velocity, and Rupert et al's test data (1997), Sorensen et al's test data (1999) and Yarin et al's test data (2000) are adopted to check the residual velocity. An example of a 500-ton patrol-boat's hull and bulkhead is also discussed. The results of the verification are good in terms of agreement, and impact velocities of this study varies from 1500 m/s to 2000 m/s, the ratio of L/D ranges from 6 to 10 and the ratio (h/D) varies from 2.0 to 11.0, may serve as a useful reference for designers.

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1. Introduction

The field of impact dynamics covers an extremely wide range of situations and is of interest to engineers from a number of different disciplines. The study of ballistic impact effects has been developed into an extensive body of knowledge under the sponsorship of military and space groups throughout the world. Generally, the ship survivability evaluation consider two parts: (1) Air-delivered weapons, such as blast (external or internal depending on fusing), fragments, shaped charge and secondary effects (fire, flooding). (2) Water-delivered weapons, such as shock, hull whipping and ship holing and flooding. The scenes for antiship threats and effects are shown in Fig. 1. In fact, the ballistic protection is one of the vulnerability reductions in requirements driving the choice of standards for marine design. Therefore, in a preliminary design, choosing a method to predict the ballistic performance of the hull and bulkhead for fragment effects is very important, based on consideration of convenience, fast, efficient performance and feasibility. The purpose of this paper is to develop a simple tool based on experimental data and semi-empirical theories to predict the ballistic performance of a structure.

The problem of predicting the complex interaction of a high-velocity projectile with geologic and structural targets has been of military interest for many years. A number of procedures have been developed over the past 200 years for making such predictions, and they generally fall into three categories: (1) Empirical approaches, in which algebraic equations are formulated based on correlation with large numbers of experimental data points, which are used to guide further experiments. (2) Analytical approaches, which concentrate on one or more aspects of the problem (such as plugging, petalling, spall, crater formation, etc.) by introducing simplifying assumptions into the full-field equations of continuum mechanics. Their solution is then attempted. Frequently, in the course of this attempt, additional simplifications must be introduced to render the problem analytically tractable. (3) Numerical approaches, in which numerical techniques are used to directly attack the full-field equations of continuum mechanics. These methods, based on finite difference or finite element techniques, have greater flexibility and applicability than do the various analytical approximations and can accurately model transient phenomena. They are still approximate in nature (one solves a set of discrete equations rather than the corresponding differential equations), but at present, errors associated with characterization of the high strain rate behavior of materials are usually far greater than errors inherent in the numerical method (Backman and Goldsmith, 1978; Goldsmith, 1999; Zukas, 1990). The following paragraphs will summarize the above three basic approaches.

Nomenclature

A	presented area of the fragment,in^2;
A_p	fragment area presented to target, in^2;
A_o	constant defined in Table 4;
a	constants (see Table 5);
B	explosive constant (see Table 1);
b	constant (see Table 5);
C_D	drag coefficient, 0.6 for primary fragments;
c	constant (see Table 5);
d	fragment diameter, in;
d_i	average inside diameter of casing, in;
$(2E')^{1/2}$	Gurney energy constant (see Table 3), ft/s;
M_A	fragment distribution factor;
m	constant defined in Table 4;
N	nose shape factor;
N_f	number of fragments with weight greater thanW_f;
N_T	total number of fragments;
n	constant defined in Table 4;
n_1	caliber radius of the target ogive of the assumed fragment;
k_v	velocity decay coefficient calculated using Eq.(11);
R_f	distance from the center of detonation, ft;
T	target thickness, in;
t_c	average casing thickness, in;
V_l	ballistic limit velocity, ft/s;
V_o	initial velocity of fragments, ft/s, other formula for various geometries (see Table 2);
V_r	residual velocity, ft/s;
V_s	striking velocity, ft/s;
v_s	fragment velocity at a distance$R_f$from the center of detonation, ft/s;
W	design charge weight, oz;
W_{ACT}	actual quantity of explosive, oz;
W_c	casing weight, lb;
$\underline{W}_f$	design fragment weight, oz;
$\bar{W}_f$	average fragment weight, oz;
W_s	fragment weight, lb,W_s=16W_f,(W_f, unit:oz);
x	parameter, $x = \frac{V_s}{V_l} - 1$;
γ	density of the target plate, lb/in^3;
ρ_a	specific density of air, (4.438x10^{-5} lb/in^3);
ρ_d	caliber density;

θ	specified angle of obliquity, degrees;
A/W_f	fragment form factor;
L/D	ratio of length to diameter of fragment.

Table 1
Mott scaling constants for mild steel casings and various explosives (Baker et al., 1980; AD-A243 272 Army TM5-1300, 1990)

Explosive	B($oz^{1/2}in.^{-7/6}$)
Baratol	0.512
Composition A-3	0.220
Composition B	0.222
Cyclotol (75/25)	0.197
H-6	0.276
HBX-1	0.256
HBX-3	0.323
Pentolite (50/50)	0.248
PTX-1	0.222
PTX-2	0.227
RDX	0.212
Tetryl	0.272
TNT	0.312

1.1. Empirical approach

In his empirical approach, Wilkins (1978) set the experimental ballistic limit velocities V_{BL} for steel targets (SAE 4340, R_C = 55) at normal incidence; Woodward (1978) studied penetration of metal targets by conical projectiles. In 1980 the ballistic limit velocity for compact fragments striking mild steel targets was estimated, and the residual velocity of perforation of mild steel targets was predicted in the paper “ A manual for the prediction of blast and fragment loadings on structures” by Baker and Williams, 1987. Neilson, 1985 described the empirical equation for perforation of mild steel plates and compared test data with the SRI equation and BRL formula. Baker and Williams, 1987 and Hohler and Stilp, 1987 studied the hypervelocity penetration of plate targets by a rod. Wilson et al., 1989 presented experimental rod impact results for 2024 aluminium targets and discussed the crater depth.

Crouch et al., 1990 experimentally tested 2024-T351 aluminium alloy plate. Charters et al., 1990 studied the penetration dynamics of rods based on direct ballistic tests of advanced armor components at 2–3 km/s for SAE 4340 R_C = 37 and formulated an empirical equation. Iglseder and Isenbergs, 1990 discussed the crater morphology at impact velocities between 8 and 17 km/s for gold targets.

Dikshit and Sundararajan, 1992 studied the penetration of thick steel plates by ogive shaped projectiles—experiment and analysis. They used the rolled homo-

Table 2
Initial Velocity of Primary Fragments for Various Geometries (Baker et al., 1980; AD-A243 272 Army TM5-1300,1990)[a]

Type	Cross-section shape	Initial fragment velocity	Maximum v_o
Cylinder		$\sqrt{2E'}\left[\dfrac{\frac{W}{W_c}}{1+\frac{W}{2W_C}}\right]^{1/2}$	$\sqrt{2E'}\sqrt{2}$
Sphere		$\sqrt{2E'}\left[\dfrac{\frac{W}{W_c}}{1+\frac{3W}{5W_C}}\right]^{1/2}$	$\sqrt{2E'}\sqrt{\frac{1}{2}}$
Steel cored cylinder	$\sqrt{2E'}\left[\dfrac{\frac{W}{W_c}}{1+\frac{(3+a)W}{6(1+a)W_c}}\right]^{1/2}$	where $a = \dfrac{d_{co}}{1.6d_i}$	$\sqrt{2E'}\sqrt{\dfrac{6(1+a)}{(3+a)}}$
Plate		$\sqrt{2E'}\left[\dfrac{\frac{3W}{5W_c}}{1+\frac{W}{5W_c}+\frac{4W_c}{5W}}\right]^{1/2}$	$\sqrt{2E'}\sqrt{3}$
Sandwich plates		if $W_{c1} = W_{c2}$, $\sqrt{2E'}\left[\dfrac{W}{W_{c1}+W_{c2}+\frac{W}{3}(1-g+g^2)}\right]^{1/2}$ where $g = \dfrac{W_{c1}+\frac{W}{2}}{W_{c2}+\frac{W}{2}}$ if $W_{c1} = W_{c2} = W_c$, $\sqrt{2E'}\left[\dfrac{\frac{W}{2W_c}}{1+\frac{W}{6W_c}}\right]^{1/2}$	$\sqrt{2E'}\sqrt{3}$

[a] W:Explosive weight; d_i,d_{co}:in; W_c:Casing weight; V_o,$\sqrt{2E'}$:ft/sec; E':Gurney energy; W,W_c,W_{co},W_{c1},W_{c2}:lbs

Table 3
Gurney energies of for various explosives (Baker et al., 1980; AD-A243 272 Army TM5-1300,1990)

Specific Weight,lb/in^3	Explosive	$\sqrt{2E'}$,ft/sec
0.0639	RDX	9600
0.0578	Composition C-3	8800
0.0588	TNT	8000
0.0621	Tritonal	7600
0.0621	Composition B(RDX/TNT)	9100
0.0682	HMX	9750
0.0664	PBX-9404	9500
0.0585	Tetryl	8200
0.0581	TACOT	700
0.0411	Nitromethane	7900
0.0635	PETN	9600
0.0527	EL506D	8200
0.0563	EL506L	7200
0.0397	Trimonite No. 1	3400

Table 4
Empirical constants for predicting the compact fragment limit velocity for mild steel targets (Baker et al., 1980)

L/D	$\frac{t}{R\sqrt{A_P}}$ [a.]	A_O	m	n
≤5	0≤0.46	1414	0.295	0.910
≤5	0.46≤1.06	1936	0.096	1.310
≤5	≤1.06	2039	0.064	0.430
> 5		1261	0.427	0.647

[a.] R is the perforation factor for perforated plates (Baker et al., 1980).

Table 5
Empirical constants for predicting the compact fragment residual velocity for mild steel targets(Baker et al., 1980)

Constant	$L/D < 5$	$L/D \geq 5$
a	1.12	1.10
b	0.52	0.80
c	1.29	1.45

geneous armour (RHA) plates with a thickness of 20, 40 and 80 mm for targets. The hardness of the steel plates (BHN) was 320, 300, 290 for 20, 40 and 80 mm thick plates, respectively, and it was experimentally measured over a range of impact velocities ($V = 300$–800 m/s) in their tests. Gupta and Madhu, 1992 presented results

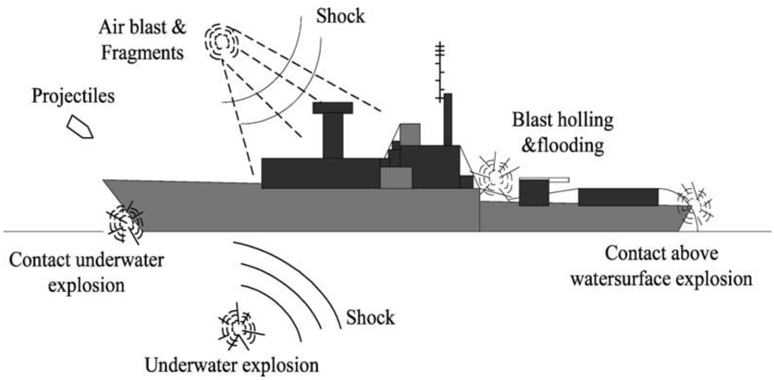

Fig. 1. Scene for antiship threats and effects.

of normal and oblique impact of a spinning armour piercing projectile on mild steel plates with thicknesses varying from 10 to 25 mm. The projectiles were fired using a standard rifle and their impact velocity was about 820 m/s in all the tests. The Vickers hardness of these plates was 142 and the yield stress was 327 MPa.

Trucano and Grady, 1995 described an experimental technique and presented results obtained from a series of experiments which used copper spheres, traveling from 2.0 to over 4.0 km/s and impacting hydrocarbon foam targets with a density of 176 kg/m^3.

Almohandes et al., 1996 experimentally investigated the ballistic resistance of steel—fiberglass reinforced polyester laminated plates. First, single mild steel plates, 1–8 mm thick, were tested, and the effects of the thickness and mechanical properties of the plate material were explored. Secondly, in-contact laminate comprising an 8 mm-thick target and spaced laminate of the same total steel thickness, with spacing distances which equal to or multiples of the bullet core diameter (6 mm), were tested, and the effects of the number, thickness, and arrangement of laminate were studied.

Edwards and Mathewson, 1997 studied the ballistic properties of tool steel for potential use as an improvised armour plate. They used the tool steel and bullets fired at the plates, hardened to 510 and 710 HV, to produce gross cracking, whether in 10 mm or 5 mm thick plates. The V_{50} values for the unwelded plates and welded plates were given for several values of Vickers hardness. Gupta and Madhu, 1997 conducted an experimental study on normal and oblique impact of hard-core projectiles on single and layered plates of mild steel, RHA steel and aluminium. The projectiles were fired at an impact velocity of 800–880 m/s. They used plates with thickness in the range 4.7–40 mm. Rupert et al., 1997 studied the energy partitioning and microstructural properties related to perforation of titanium and steel targets.

Piekutowski et al., 1999 performed a series of depth-of-penetration experiments using 7.11 mm diameter, 71.12 mm long, ogive-nose steel projectiles and 254 mm diameter, 6061-T6511 aluminum targets with striking velocities between 0.5 and 3.0 km/s. They showed good agreement between the rigid-projectile penetration data and a cavity-expansion model. Sorensen et al., 1999 conducted a computational study to

assess terminal ballistic performance when a steel sheath, or jacket, was added to a depleted uranium penetrator striking semi-infinite rolled homogeneous armor (RHA). Borvik et al., 1999 discussed the ballistic penetration of steel plates and presented a research programme in progress, the main objective of which was to study the behaviour of Weldox 460 E steel plates impacted by blunt-nosed cylindrical projectiles in the lower ordnance velocity regime.

Yarin et al., 2000 described the fracturing of targets and projectiles during normal penetration using a model of chaotic disintegration modifying the theory of chaotic disintegration of liquids

1.2. Analytical approach

There have also been many studies on the analytical approach to ballistic impact modeling. Thomson, 1955 developed an approximated theory of armour penetration from a quasi-dynamical approach using the conical and the ogival head on thin plates. Zaid and Paul, 1959 studied oblique perforation of a thin plate by a truncated conical projectile with high velocity. Awerbuch, 1970 presented a mathematical model which described the mechanism involved in the normal penetration of metallic targets. Wilkins, 1978 studied the mechanics of penetration and perforation and identified the important material parameters of targets and projectiles that influence perforation. Jonas and Zukas, 1978 reviewed available analytical methods for the study of kinetic energy (inert) projectile armor interactions at ordnance velocities (0.5-2 km/s). Jones et al., 1980 studied ballistics calculations of R.W. Gurney and reviewed Gurney's original method for estimating the terminal velocity of fragments accelerated by symmetrical explosive systems. Luk and Piekutowski, 1991 developed an analytical model to predict the performance of eroding long rods that penetrated metallic targets at normal incidence with impact velocities above 2.0 km/s. Wijk (1999) studied high-velocity projectile penetration into thick armour targets.

1.3. Numerical approach

Numerical techniques may be thought of as higher order methods of analysis and require a detailed description of the material constitutive behaviour over the range of strain rates being studied, accompanied by extensive characterization. The more complete the description of the material behaviour, the more realistic the results of the simulation, and the more complex, expensive, and time consuming the requirements for material characterization. The discretization techniques of the numerical methods most commonly used are the finite-difference and finite-element techniques. The bulk of the computer codes used on a production basis for impact studies fall into two categories: Lagrangian codes and Eulerian codes. Lagrangian codes follow the motion of fixed elements of mass, while the computational grid is fixed in the material and distorts with it. For problems in which large distortions predominate or where mixing of materials initially separated occurs, a Eulerian description of the material behaviour is necessary. In the Eulerian approach, the computational grid is fixed in space while material passes through it. Some of the works which have

applied computer codes to impact problems are reviewed here. Sedgwick et al., 1978 presented a numerical investigation into the penetration mechanics using the continuum mechanics computer code HELP (Hydrodynamic Elastic Plastic) and compared the computational results with experimental data. Anderson et al. (1995) published a paper on the velocity dependence of the *L*/*D* effect for long-rod penetrators using the nonlinear, large deformation Eulerian wavecode CTH. Westerling et al., 1997 studied the penetration into alumina targets of homogeneous, segmented and telescopic tungsten projectiles through numerical simulations using AUTODYN-2D code and experimental tests. Belingardi et al., 1998 presented numerical simulation of the fragmentation of composite material plates due to impact using DYNA3D explicit finite element code.

It is very desirable, then, to have simple empirical or theoretical expressions to describe the impact process, for use in systems design and tradeoff studies. Unfortunately, the impact process inherently involves the trajectory as shown in Fig. 2, different types of projectiles as shown in Fig. 3, complex large multidimensional deformations, and the material responses, which are strongly nonlinear. Therefore, theoretical treatment short of integration of the complete partial differential equations governing the process is restricted to a few special cases, and usually involves a number of undetermined parameters which must be determined empirically. Such theories either appeal to gross momentum and energy arguments, or make sweeping, simplifying assumptions regarding the form of the motion resulting from impact in

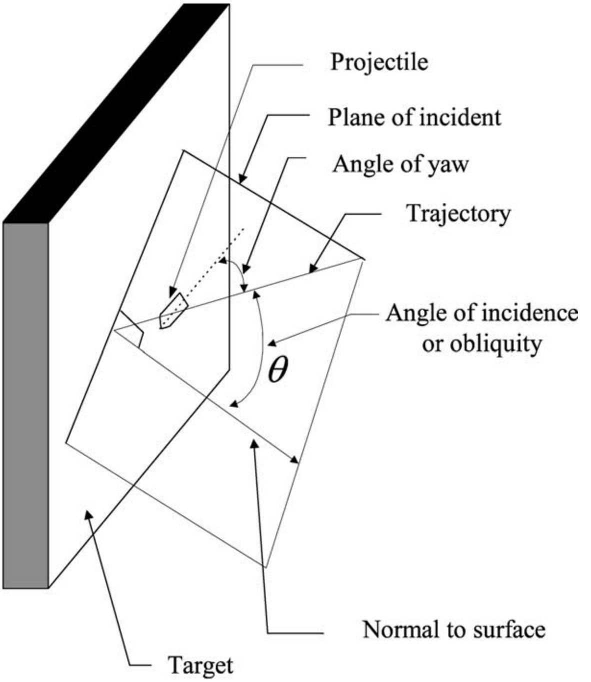

Fig. 2. Scheme of impact.

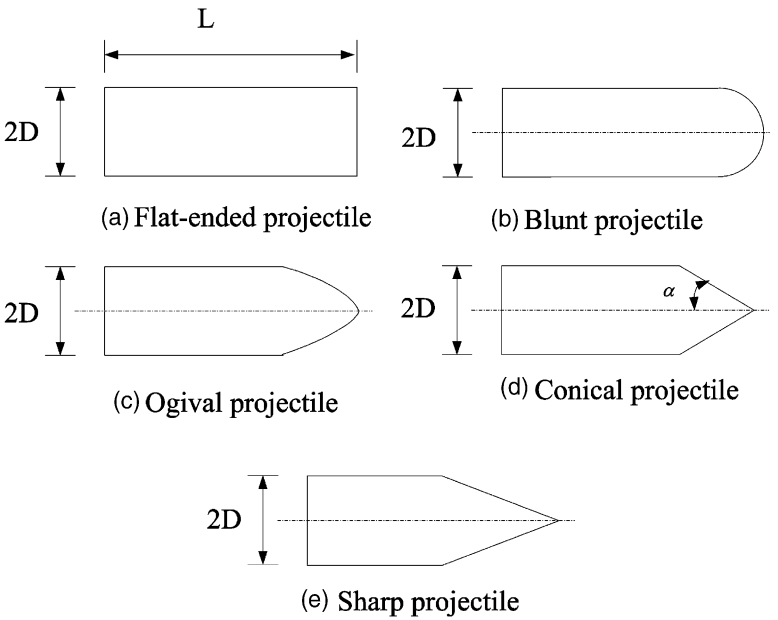

Fig. 3. Types of projectiles.

order to reduce the dimensionality of the problem. However, when properly normalized, either to experimental data, beyond the experimental range, or to detailed numerical solutions, they along with purely empirical fits can provide simple and useful expressions for use in systems studies within their ranges of validity.

Therefore, in a preliminary design, it is important to choose a method which is simple, quick and feasible for predicting the performance of protective structures. In particular, study of the performance of perforation of special bulkheads and shell plating of navy ships, the outside structures of armoured tanks, and the structures of casing boxes for launch missiles are important in the preliminary design stage. This paper presents an empirical method coupled with test data to predict performance of the perforation of protective structures, in which the striking velocity is calculated using Gurney formulas, and the residual velocity is estimated using Baker et. al.'s equation, (1980). The ballistic limit velocity of mild steel is verified using Woodward's tests (1978) and Edwards and Mathewson's experiments (1997). The residual velocity of mild steel is also verified using Rupert et al.'s tests (1997), Sorensen et al.'s tests (1999) and Yarin et al.'s tests (2000), and the results all agree. The results of this study may serve as a useful reference for designers.

2. Semi-empirical method

Empirical relations used in penetration mechanics are most useful when the number of variables being correlated is small. In this semi-empirical method, algebraic

equations are formulated, based on correlation with large numbers of experimental data points, and these are used to guide further experiments. Such efforts are often closely related to tests performed to discriminate between the performance characteristics of various materials or structures for a particular design objective.

In this paper, a semi-empirical method is established to predict the resistant performance of perforation for protective structures. The procedure is shown in Fig. 4, which includes four stages. The first stage is the basic data stage: the material property of the target, the geometry of the target, the target thickness and the types of explosives, the material in the casing, and the shapes of the explosives are prepared. The second stage is the impact stage: the fragment distribution, the mass and shape of the fragments, the initial velocity, the striking velocity, and the incident angle are calculated. The third stage is the penetration stage: the depth of penetration, the ballistic limit velocity, the trajectory of the projectile toward the target, and the failure mode are evaluated. The fourth stage is the perforation stage: a comparison of the penetration and thickness of the target is made, and the residual velocity is estimated. In the procedure, the mass distribution of fragments is obtained using Mott's equation (1963), the initial and striking velocities are calculated using Gurney's formulas and residual velocity is estimated using Baker et al.'s equation (1980). The calculation of the fragment weight, the initial velocity of the fragment, the striking velocity, the penetration and the residual velocity is presented in the following paragraphs.

2.1. Fragment weight

The fragment weight can be used to calculate the ballistic velocity. Upon detonation of an encased explosive, the casing breaks up into a large number of fragments with varying weights and velocities, and the destructive potential of these fragments to humans, vehicles and protective structures is a function of their kinetic energy distribution. Therefore, through testing or analysis the velocity and weight of the 'worst case' fragment must be determined and utilized as a design criterion for fragment shields. In general, consideration of the impact effects of fragments when designing protective structures that can resist the effects of conventional weapons is most important. Because the Mott equation (1963) yields estimates of the fragment mass distribution resulting from the detonation of evenly distributed explosives within a uniform thickness cylindrical casing for naturally fragmenting casings, this paper adopts the Mott equation to calculate the fragment mass distribution. The number of fragments produced by cylindrical cased charge weighing more than a given design fragment is written as

$$N_\mathrm{f} = \frac{8 \cdot W_\mathrm{c} \cdot e^{-(W_f^{1/2}/M_A)}}{M_\mathrm{A}^2} \tag{1}$$

and

$$M_\mathrm{A} = B \cdot (t_\mathrm{c})^{5/6} \cdot (d_\mathrm{i})^{1/3} \cdot (1 + t_\mathrm{c}/d_\mathrm{i}), \tag{2}$$

The largest fragment produced by an explosion can be found by setting $N_\mathrm{f} = 1$. Thus, the weight of the largest fragment from Eq. (1) is given by

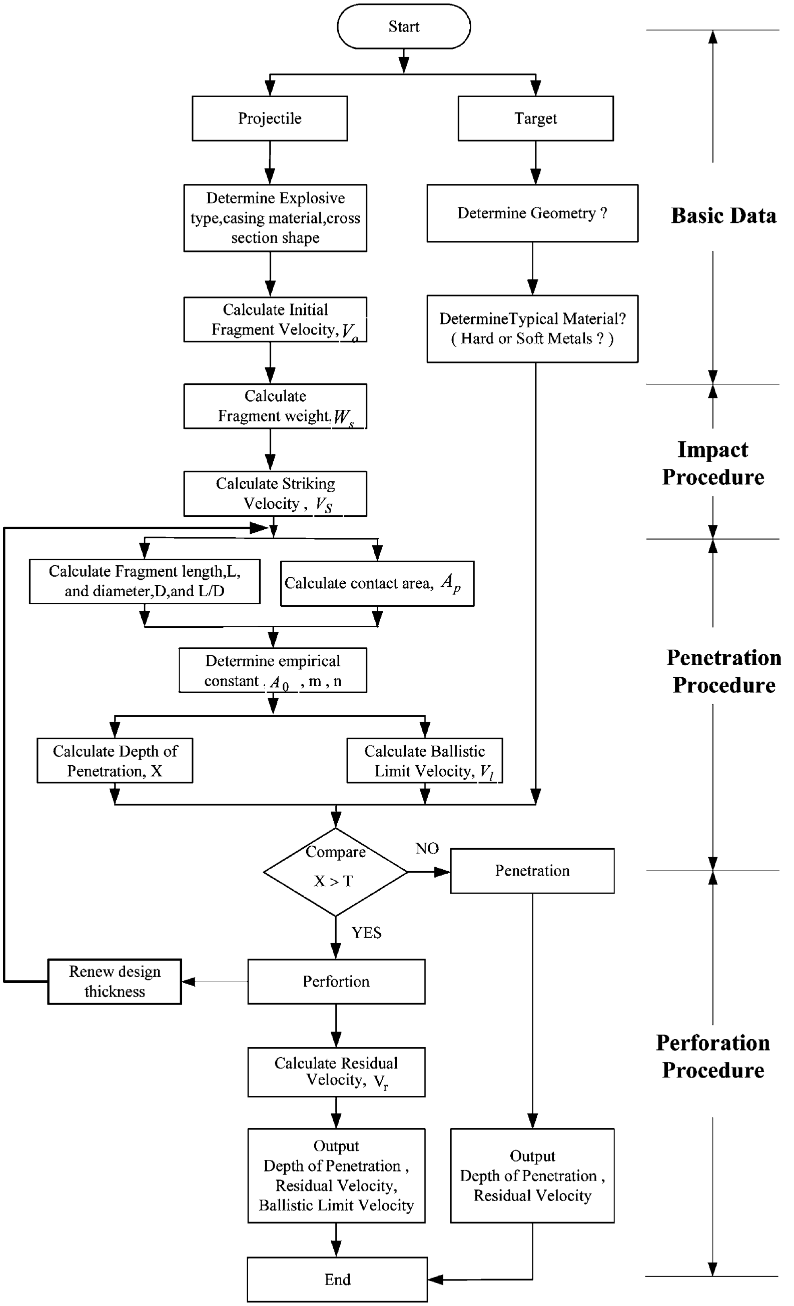

Fig. 4. Flow chart of the semi-empirical method.

$$W_f = [M_A \cdot \ln(8 \cdot W_c / M_A^2)]^2. \tag{3}$$

If the lowest design fragment weight produced by an explosion is equal to zero, then the following expression for the total number of fragments is obtained:

$$N_T = 8 \cdot W_c / M_A^2, \tag{4}$$

Hence, the average particle weight can be found:

$$\overline{W}_f = 16 \cdot W_c / N_T \tag{5}$$

In order to determine the damage potential of primary fragments, it is necessary to evaluate the caliber density and shape of the fragments. The influence of the fragment weight on the fragment diameter ratio is expressed in terms of the caliber densityρ_d of the fragment, which is defined as

$$\rho_d = W_f / d^3 \tag{6}$$

The nose shape factor N is defined as follows:

$$N = 0.72 + 0.25 \cdot \sqrt{n_1 - 0.25}. \tag{7}$$

2.2. Initial fragment velocity

The Gurney model yields explicit algebraic relationships for estimating the velocity imparted to a metal in contact with a detonating explosive. Thus, the Gurney method can be directly and simply applied to a multitude of explosive-metal interaction problems. Design and parametric studies can be readily performed. Applications of the Gurney model include warhead and fragmentation design, the study of the efficiency of conversion of high explosive chemical energy to kinetic energy of a plate, explosive initiation by impact of an explosively driven plate, the launching of a dielectric plate by electrical detonation of a metal foil, and calculation of the speeds of layers of metal fragments in conjunction with shock wave physics (Mott, 1963).

The so-called Gurney formulas are very successful in determining terminal fragment velocities. These formulas are derived on the basis of three fundamental assumptions: (1) The chemical energy of the explosive is converted into kinetic energy of fragments and detonation products. (2) The velocity distribution of the detonation products varies linearly from zero to the velocity of the fragment. (3) The density of the products is uniform.

Therefore, the most common technique for calculating the initial velocity of fragments in contact with an explosive charge is the Gurney method (1980). In this paper, the initial velocity of primary fragments resulting from the detonation of a cased explosive is a function of the explosive output and the ratio of the explosive charge weight to casing weight. Therefore, this paper adopts Gurney method to calculate the initial velocity of primary fragments resulting from detonation of a cylindrical casing with evenly distributed explosives and an initial fragment velocity expressed as

$$V_o = (2E')^{1/2} \left(\frac{W/W_c}{1 + 0.5 W/W_c} \right)^{1/2} \tag{8}$$

Applying a 20% factor of safety, the design charge weight is

$$W = 1.2 \cdot W_{\mathrm{ACT}} \tag{9}$$

2.3. Striking velocity

The striking velocity (impact velocity) V_s is one of the most important parameters commonly used in penetration mechanics. Thus, when an explosion is located close to an object (an acceptor explosive or barrier), the striking velocity V_s at which a fragment strikes the object is approximately equal to the initial velocity V_o. However, if the detonation is located at a relatively large distance from the object, then the impact or striking velocity of the fragment may be substantially less than its velocity immediately after the explosion. This variation in velocity, which is primarily a result of air resistance, is also a function of the physical properties of the casing and the distance between the donor explosive and the object in Ref. (AD-A243 272 Army TM5-1300, 1990).

When the protective barriers are located 20 feet or less from a detonation in Ref. (AD-A243 272 Army TM5-1300, 1990), the variation between the striking and initial velocities usually can be neglected. On the other hand, to determine the effects of primary fragment impact on structures further away from a detonation, the variation of the fragment velocity with distance should be included in the design. The striking velocity is a function of the initial velocity in Gurney formulas, the explosive weight, the air density, the geometry size of the fragment and distance in Ref. (AD-A243 272 Army TM5-1300, 1990). In addition, the striking velocity is decay with exponent. Finally, the striking velocity reaches a constant value. In this paper, the fragment velocity of major concern is the velocity with which the 'design fragment(s)'(the worst case fragment(s) which the structure must be designed to withstand) strikes the protective structure, where the flight phenomenon of the fragment is similar to the description in Ref. (AD-A243 272 Army TM5-1300, 1990). Therefore, this paper adopts calculation of the striking velocity in Ref. (AD-A243 272 Army TM5-1300, 1990), expressed as

$$V_s = V_0 \cdot e^{-[\,12 \cdot k_{\mathrm{v}} \cdot R_{\mathrm{f}}\,]} \tag{10}$$

and

$$k_{\mathrm{v}} = (A/W_{\mathrm{f}}) \cdot \rho_{\mathrm{a}} \cdot C_{\mathrm{D}} \tag{11}$$

2.4. Depth of penetration

According to the MIL-STD-662F, the ballistic limit velocity is the minimum velocity at which a particular projectile is expected to consistently, completely penetrate armour of a given thickness with given physical properties at a specified angle of obliquity. The empirical equation of the ballistic limit velocity derived by Baker et al., 1980 is based on parameter studies on the fragment size, fragment weight, target thickness and angle of obliquity, by means of conservation of energy, momentum

and mass, and finally based on correlation with large numbers of experimental data points, which are used to guide further experiments. These equations are more applicable to engineering problems. Therefore, this paper adopts Baker et al.'s equation to calculate the ballistic limit velocity for compact fragments striking mild steel targets, which can be estimated as (Baker et al., 1980)

$$V_l = \frac{A_0}{\sqrt{W_s}} \cdot A_P^m \cdot (T \cdot \sec\theta)^n \tag{12}$$

According to the definition of the ballistic limit velocity in Eq. (12), when the ballistic limit velocity V_l is equal to the striking velocity V_s, the depth of penetration T' for the target is derived as follows:

$$T' = \cos\vartheta \cdot \left(\frac{V_s \cdot \sqrt{W_s}}{A_o \cdot A_p^m}\right)^{1/n} \tag{13}$$

where θ, L/D, A_p, W_s, A_0, m and n are as previously defined in Eq.(12).

2.5. Residual velocity

In general, the experiment measure of the residual velocity is one of the most accurate methods for obtaining the residual velocity. Baker et al. (1980) derived the empirical equation of the residual velocity based on the ballistic limit velocity, striking velocity, target thickness and fragment weight, coupled with experiment data. The empirical equation is more applicable to engineering problems. In this paper, because we seek to solve a practical engineering problem, the residual velocity of a fragment that has perforated a mild steel plate is estimated as follows (Baker et al., 1980):

$$V_r = V_l \cdot \beta \left[\frac{a \cdot x^2 + b \cdot x + c \cdot \sqrt{x}}{x + 1}\right], \tag{14}$$

where (a) $L/D \leq 2$

$$\beta = \frac{1}{(1 + \gamma \cdot A_p \cdot T/W_s)^{1/2}},$$

(b) $L/D > 2$, $\beta = 1$

3. Verification and numerical analysis

In this paper, Woodward's tests (1978) and Edwards and Mathewson's experiments (1997) are used to verify the ballistic limit velocity, and Rupert et al.'s tests (1997), Sorensen et al.'s tests (1999) and Yarin et al.'s tests (2000) are adopted to check the residual velocity at normal impact with steel-flat end projectile, which was

Table 6
Projectile dimensions for Woodward's test data (1978)[a]

Cone included angle, α	Maximum tip flat(mm)	Diameter, D (mm)	Total length, L(mm)	Mass, M (gm)
45°	0.015	4.76	25.4	2.83

[a] M=W/g, W: weight, g: acceleration of gravity

calculated using the present semi-empirical method. The verifications and numerical analysis are discussed in the follow paragraphs.

3.1. *Verification of the ballistic limit velocity*

Woodward's tests (1978) and Edwards and Mathewson's experiments (1997) were adopted to verify the ballistic limit velocity calculated using the present semi-empirical method.

3.1.1. *Woodward's tests*

In Woodward's experiment, the projectile has a mass of 2.83 gm, a length (L) of 25.4 mm and a diameter (D) of 4.76 mm. The important dimensions of the projectile are given in Table 6. Two different thicknesses (h) of mild steel plates, 1.60 mm and 4.78 mm, are adopted for target plates. The ratio of the projectile length to the diameter (L/D) is 5.34, and the ratios of the plate thickness to the projectile diameter (h/D) are 0.33, 1.0, respectively. These plates are thin targets based on Goldsmith's definition (1999). The target materials have a strength parameter of σ_0 (= 976 MPa.) and a Vickers Hardness (= 136 HVS), as given in Table 7.

The computed values obtained using the present semi-empirical method and Woodward's test data (1978) are given in Table 8. It is obviously found that the ballistic limit velocities of the two different plates, 1.60 and 4.78 mm thick, calculated using the present semi-empirical method to calculate, are 176 and 357 m/s, respectively, and Woodward's tests are 202 and 375 m/s, respectively. The error of the computed values and Woodward's tests is 12.9% in thickness of 1.60 mm and another is 5.0% in thickness of 4.78 mm.

Table 7
Target materials for Woodward's test data (1978)

Material	Thickness (mm)	Vickers hardness (HVS)	σ_0 (MPa)	n
Mild steel	1.60	117	563	0.163
Mild steel	4.78	136	976	0.263

Table 8
Verification of the ballistic limit velocity using Woodward's test data

Target	L/D[a.]	h/D[a.]	Present method (m/s) (A)	Woodward's test (m/s) (B)	Error %[b.]
1.6 mm	5.34	0.333	176	202	12.9%
4.78 mm	5.34	1.0	357	375	5%

[a.] L/D is the projectile length to diameter ratio, and h/D is the plate thickness to projectile diameter ratio

[b.] Error$=\frac{|(A)-(B)|}{(B)}$ %

Table 9
Projectile dimensions for Edwards's test data (1997)

Cone included angle	Diameter, D (mm)	Total length, L (mm)	Mass, M (gm)
60°	5.59	20.15	5.91

3.1.2. *Edwards et al.'s tests*

In Edwards et al.'s tests, the 7.62 mm bullet had a core mass of 5.91 gm, a core length of 20.15 mm, and a core diameter of 5.59 mm as given in Table 9. The ratio of the projectile length to the diameter (L/D) was 3.6. The ball round consisted of a fine grained ferrite-pearlite steel core (the composition (weight percent) of which was C 0.13, Si 0.22, Mn 0.48, S 0.018, P 0.012, balance Fe) with hardness of 236 HV, surrounded by a low carbon steel jacket, coated with gilding metal and supported in the jacket by a lead filling. The angle of the ogival nose was 30°, but the steel penetrator was flat fronted. The target plate was 5 mm thick mild steel, and the composition (weight percent) was C 0.19, Mn 1.02, Si 0.03, $S < 0.005$, P 0.015 and balance Fe with hardness of 133 HV as shown in Table 10. The ratio of the plate thickness to the projectile diameter (h/D) was 0.9. The plate was a thin target based on Goldsmith's definition (1999).

The computed values obtained using the present semi-empirical method and

Table 10
Target materials for Edwards' test data (1997)

Material	Thickness (mm)	Vickers hardness (HVS)	Composition (weight per cent)
Mild steel	5	133	C:0.19;Mn:1.02;Si:0.03;S$<$0.005;P:0.015

Edwards et al.'s test data are given in Table 11. From Table 11, it is clearly seen that the ballistic limit velocity of the target plate, with 5.00 mm thickness, obtained using the present method, is 451 m/s, and that obtained using Edwards et al.'s test data is 466 m/s; the error is about 3.2%.

3.2. Verification of residual velocity

Rupert et al.'s tests (1997), Sorensen et al.'s tests (1999) and Yarin et al.'s experiments (2000) are adopted to check the residual velocity calculated using the present semi-empirical method.

3.2.1. Rupert et al.'s tests

In Rupert et al.'s tests (1997), a rod projectile used in the impact experiments had a mass of 65 gm, a diameter of 7.82 mm, a length-to-diameter ratio of 10 (L/D), and hemispherical nose shape. Four slightly different plate thicknesses were used for the RHA (i.e., rolled homogeneous armor) targets: two at 44.69 mm with an areal density of 350.8 kg/m^2 and two at nominal 40 mm with a corresponding areal density of 307.3 kg/m^2. The penetrator was fired from a laboratory gun at nominal 1500 m/s and 2000 m/s impact velocities. The ratio (h/D) of the target thickness to the projectile diameter was from 5 to 6. The targets were intermediate thickness plates based on Goldsmith's definition (1999).

Table 12 shows that the residual velocities calculated using the present semi-empirical method are 1355 m/s for 44.69 mm thickness, 1358 m/s for 44.69 mm thickness, 1845 m/s for 40.03 mm thickness and 1870 m/s for 39.91 mm thickness, when impact velocities are 1507, 1510, 1973 and 1997 m/s, respectively, and that those obtained using Rupert et al.'s test are 1271, 1275, 1904 and 1931 m/s. The residual velocity error ranges from 3.2 to 6.7 %, the average residual velocity error is 4.85%, and the residual velocity error decreases gradually from 6.7 to 3.1%, when the impact velocities gradually increase from 1500 m/s to 2000 m/s for intermediate targets and for the ratio L/D (= 10).

3.2.2. Sorensen et al.'s tests

In Sorensen et al.'s tests (1999), a long rod projectile had a mass of 1.777 kg, a diameter of 23.0 mm and a length-to-diameter ratio of 10 (L/D), and the impact

Table 11
Verification of the Ballistic limit velocity using Edwards's test data

Target	L/D[a.]	h/D[a.]	Present method (m/s) (A)	Edwards's test (m/s) (C)	Error %[b.]
5.0 mm	3.6	0.9	451	466	3.2%

[a.] L/D is the projectile length to diameter ratio, and h/D is the plate thickness to projectile diameter ratio

[b.] Error$=\dfrac{|(A)-(C)|}{(C)}$ %

Table 12
Verification of the residual velocities using Rupert et al.'s test data

Target thickness (mm)	Impact velocity V_s (m/s)	h/D[a.]	L/D[a.]	Present method V_r (m/s) (A)	Rupert et al.'s test V_r(m/s) (D)	Error %[b.]
44.69	1507	5.71	10	1355	1271	6.7%
44.69	1510	5.71	10	1358	1275	6.4%
40.03	1973	5.12	10	1845	1904	3.1%
39.91	1997	5.10	10	1870	1931	3.2%

[a.] L/D is the projectile length to diameter ratio, and h/D is the plate thickness to projectile diameter ratio

[b.] Error$=\frac{|(A)-(D)|}{(D)}$ %

velocity was 1700 m/s at normal incidence against 150 and 250 mm RHA (i.e., rolled homogeneous armor). The ratio (h/D) of the target thickness to projectile diameter ranges from about 6.5–11. The 150 mm target was an intermediate thickness plate and the 250 mm target belongs to semi-infinite plate based on Goldsmith's definition (1999).

Table 13 shows that the residual velocity error is 0.7% for a 150 mm thick target and reaches 26.4% for a 250 mm thick target, at an impact velocity 1700 m/s. It is clearly seen that the residual velocity is influenced by the ratio of h/D.

3.2.3. Yarin et al.'s tests

Yarin et al.'s tests (2000) in a normal impact experiment were performed using tungsten sinter alloy rods (D = 20 mm, L/D = 6) against 40 and 70 mm rolled homogeneous armour (RHA) at an impact velocity 1700 m/s. The ratio (h/D) of the target thickness to the projectile diameter varied from about 2–3.5.

Table 14 shows that the predicted residual velocity using the present semi-empirical method is 1603 m/s, and that that obtained using Yarin et al.'s test data is 1578 m/s for a 40 mm target, when impact velocity is 1681m/s, that the residual velocity predicted is 1556 m/s, and that that obtained using Yarin et al.'s test is 1506 m/s for a 70 mm target, when impact velocity is 1695 m/s, and that the residual velocity predicted is 1557 m/s, and that that obtained using Yarin et al.'s test is 1483 m/s for a 70 mm target, when impact velocity is 1696 m/s. The residual velocity error increases gradually from 1.6 to 5.0% when the h/D ratio increases gradually from 2.0–3.5.

4. Numerical example of a 500-ton patrol-boat's hull and bulkhead

The ballistic performance of a 500-ton littoral patrol-boat's hull and bulkhead as shown in Fig. 5 under three different velocities, 1128, 1768, and 2438 m/s, is discussed. The thickness of hull and bulkhead made of mild steel is 6 mm, 10 mm thick, relatively. The double layer target made of hull and bulkhead is also discussed.

4.1. Principal dimensions and material properties

The principal dimensions of projectile and material properties are:

1. Projectile mass: M=207±2 grains;
2. Projectile weight: W_s=0.03 lb;
3. Projectile length: L=0.623 in;
4. Projectile diameter: D=0.5 in;
5. The ratio of L/D: L/D=1.25;
6. Projectile area presented to target: A_p=0.196 in^2;
7. Specified angle of obliquity: θ=0°;
8. Impact velocity: V_1=1128 m/s, V_2=1768 m/s, V_3=2438 m/s;
9. Projectile density: γ_p=0.283 lb/in^3;

Table 13
Verification of residual velocities using Sorensen et al.'s test data

Target thickness (mm)	Impact velocity V_s (m/s)	h/D[a]	L/D[a]	Present method V_r (m/s) (A)	Sorensen et al.'s test V_r (m/s) (E)	Error %[b]
150	1700	6.52	10	1541	1530	0.7%
250	1700	10.87	10	1327	1050	26.4%

[a] L/D is the projectile length to diameter ratio, and h/D is the plate thickness to projectile diameter ratio

[b] Error$=\frac{|(A)-(E)|}{(E)}$ %

Table 14
Verification of the residual velocities using Yarin et al.'s test data

Target thickness (mm)	Impact velocity V_s (m/s)	h/D[a.]	L/D[a.]	Present method V_r (m/s)(A)	Yarin et al.'s test V_r(m/s) (F)	Error %[b.]
40	1681	2	6	1603	1578	1.6%
70	1695	3.5	6	1556	1506	3.3%
70	1696	3.5	6	1557	1483	5.0%

[a.] L/D is the projectile length to diameter ratio, and h/D is the plate thickness to projectile diameter ratio

[b.] Error=$\frac{|(A)-(F)|}{(F)}$ %

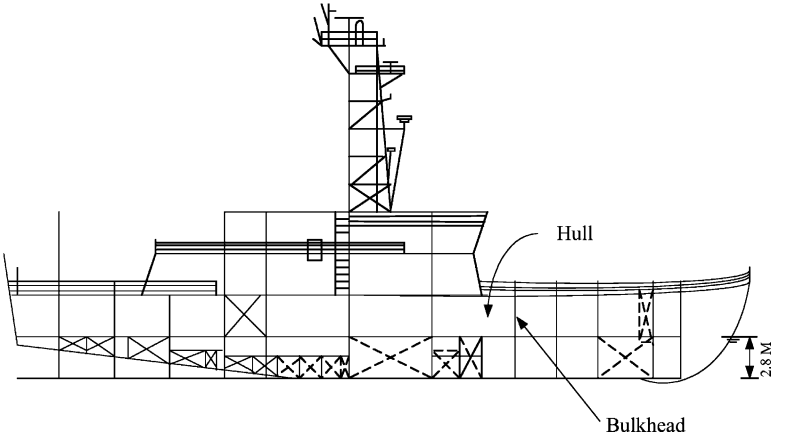

Fig. 5. Schematic diagram of the 500-ton patrol-boat.

10. Target density: γ_t=0.283 lb/in^3;
11. Hull thick: T_h=6 mm;
12. Bulkhead thick: T_b=10 mm.

4.2. Results and discussion

According to the calculation procedure used for the ballistic performance, the residual velocities of the double layer target made of hull and bulkhead are obtained, as shown in Table 15. The perforation figures are shown in Fig. 6 during impact velocity, 1128 m/s.

Table 15 shows that the residual velocities calculated using the present semi-empirical method are 546, 996, 1432 m/s for 10 mm thick (first layer: hull), and 297, 673, 1004 m/s for 6 mm thick (second layer: bulkhead), when impact velocities are 1128, 1768, 2438 m/s. Three different impact velocities can perforate the double

Table 15
Residual velocities of double layered thickness for a 500-ton patrol-boat

Projectile Mass, M g	Impact velocity, V_s m/s	Thickness of Hull (first layer),T_h mm	Residual velocity of hull, Vr_1 m/s	Thickness of Bulkhead (second layer),T_b mm	Residual velocity of bulkhead, Vr m/s	Result
13.4	1128	10	546	6	297	perforation
13.4	1768	10	996	6	673	perforation
13.4	2438	10	1432	6	1004	perforation

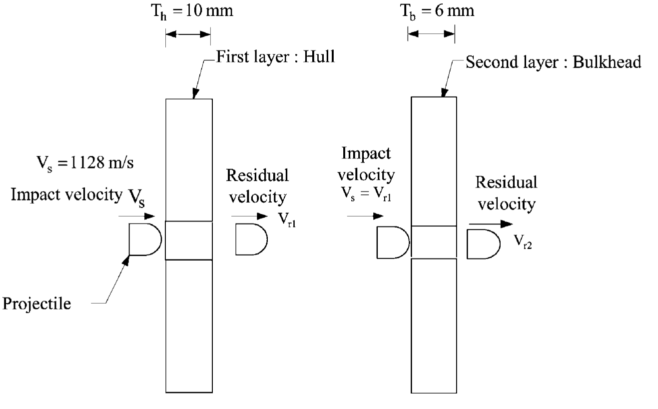

Fig. 6. Schematic diagram of Perforation for 500-ton patrol-boat.

layer target made of hull and bulkhead for a 500-ton littoral patrol-boat It is obvious that hull and bulkhead can not resist the three different impact velocities.

5. Conclusions

This paper has presented a semi-empirical method. In the calculation procedure, the mass distribution of fragments is obtained using Mott's equation, the initial and striking velocities are calculated using Gurney's formulas and the residual velocity is estimated using Baker et al.'s equation. Based on comparison with Woodward's tests (1978) and Edwards's experiments (1997) for the ballistic limit velocity, and with Rupert et al.'s tests (1997), Sorensen et al.'s tests (1999), Yarin et al.'s tests (2000) and the numerical example of a 500-ton patrol-boat's hull and bulkhead for the residual velocity, we can draw the following important conclusions.

1. The comparison of the ballistic limit velocity calculated using the present semi-empirical method with using Woodward's tests (1978) and Edwards's experiments

(1997) is good in agreement. And almost all of the errors vary from 3.2 to 12.9%. The slight error remains because the nose shape of projectile is not completely similar, the material in the target for mild steel plate is also not completely the same in compositions, strength and hardness.

2. The comparison of the residual velocity calculated using the present semi-empirical method with using Rupert et al.'s tests (1997), Sorensen et al.'s tests (1999) and Yarin et al.'s tests (2000) is good in agreement. And the errors also vary from 0.7 to 6.7%, while the error of only the 250 mm target reaches 26.4%. A small error occurs because the impact velocity and the h/D ratio are also significantly effective parameters except that the descriptions of the above reasons.
3. Three different impact velocities, 1128, 1768, 2438 m/s, can perforate the double layer target made of hull (10 mm thickness) and bulkhead (6 mm thickness) for a 500-ton littoral patrol-boat. It is obvious that hull and bulkhead cannot resist the three different impact velocities.

On the whole, the predictions obtained using the present semi-empirical method based on verification of the ballistic limit velocities and residual velocities are good in agreement with experiments, and this study that impact velocities varies from 1500 to 2000 m/s, the ratio of L/D ranges from 6 to 10 and the ratio (h/D) varies from 2.0 to 11.0, may serve as a useful reference for designers to consider the fragment effects in special bulkheads and shell plating of Navy ships. When the effects of parameters, such as the impact velocity, projectile mass, nose shape, L/D, h/D and contact area, on prediction of the ballistic performance of protective structures are studied in the future, the present semi-empirical method will be applied to a more widespread range.

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